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Residual Energy in Weak and Strong MHD Turbulence

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Abstract. Recent numerical and observational studies revealed that spectra of magnetic and velocity fluctuations in MHD turbulence have different scaling indexes. This intriguing feature has been recently explained in the case of weak MHD turbulence, that is, turbulence consisting of weakly interacting Alfvén waves. However, astrophysical turbulence is strong in majority of cases. In the present work, we propose a unifying picture that allows one to address weak and strong MHD turbulence on the same footing. We argue that magnetic and kinetic energies are different in both weak and strong MHD turbulence. Their difference, the so-called residual energy, is spontaneously generated by turbulence, it has the Fourier spectrum $E_r(k) = E_v(k) - E_b(k) \propto -f_w(k_{\parallel}/k_{\perp})k_{\perp}^{-2}$ in weak turbulence, and $E_r(k) \propto -f_s(k_{\parallel}/k_{\perp})k_{\perp}^{-3}$ in strong turbulence. Here $f_{w,s}(x)$ are functions declining fast for $x > C_{w,s}$ and not significantly varying for $x < C_{w,s}$ with some constants $C_{w,s}$, and k_{\parallel} and k_{\perp} the field-parallel and field-perpendicular wave vectors with respect to the applied strong uniform magnetic field.

1. Introduction

Magnetic fields and turbulence are common in a variety of astrophysical plasmas, from planets and stars to interstellar and intergalactic media. Magnetic turbulence is also commonly invoked to explain small-scale features of the solar wind. Numerical simulations and analytic modeling play an important role in interpreting observational data. Recently, it has been found that magnetic and velocity fluctuations are not in equipartition in MHD turbulence (e.g., Podesta et al. 2007; Tessein et al. 2009; Chen et al. 2011a; Boldyrev et al. 2011), which seems to be at odds with basic assumptions of conventional models of MHD turbulence. The goal of the present contribution is to propose an explanation for this intriguing phenomenon. In contrast with ordinary incompressible turbulence, which is always in a strongly coupled state, incompressible MHD turbulence can exhibit two distinct regimes of weak and strong turbulence. This stems from the fact that the MHD system possesses Alfvén waves that can coalesce and scatter off each other. When during a single interaction the wave amplitudes change only slightly, turbulence is weak, otherwise, it is strong. It is important to note however that even if MHD turbulence is weak at large scales, its strength increases toward small scales, so that the range of scales where weak MHD turbulence may exist is typically limited. In this contribution we present a unifying model of MHD turbulence valid for both weak and strong regimes. The incompressible MHD equations for magnetic

and velocity fields, $\mathbf{b}(\mathbf{x}, t)$ and $\mathbf{v}(\mathbf{x}, t)$, have especially useful form when written in the so-called Elsasser variables $\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{b}$,

$$\left(\frac{\partial}{\partial t} \mp \mathbf{v}_A \cdot \nabla \right) \mathbf{z}^\pm + (\mathbf{z}^\mp \cdot \nabla) \mathbf{z}^\pm = -\nabla P. \quad (1)$$

The equations are written in a frame with zero mean-flow velocity, \mathbf{b} is the fluctuating magnetic field normalized by $\sqrt{4\pi\rho_0}$, $\mathbf{v}_A = \mathbf{B}_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity corresponding to the uniform magnetic field \mathbf{B}_0 (so that the total magnetic field is $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$), $P = p/\rho_0 + b^2/2$, it includes the plasma pressure, p , and the magnetic pressure, ρ_0 is the constant mass density, and we neglect driving and dissipation terms (e.g., Biskamp 2003; Marsch & Mangeney 1987). In what follows we assume that turbulence is driven at large scales, that can be mimicked by adding forcing terms to the right-hand sides of Eqs. (1). Small-scale turbulence is expected to be independent of the large-scale driving (e.g., Mason et al. 2008). We will also assume that the uniform guide field is strong compared to the rms fluctuations, that is, $b_{rms} \ll B_0$.

2. The energy spectrum

The ideal MHD equations conserve the two Elsasser energies, $E^\pm = \int |\mathbf{z}^\pm|^2 d^3x = \int e^\pm(\mathbf{k}) d^3k$. When the energies are supplied to the system at large scales, they get redistributed over scales by nonlinear interactions, and removed from the system at small dissipation scales. One can argue that the energy gets redistributed predominantly over the modes whose wavevectors are approximately normal to the strong guide magnetic field. We will concentrate on the so-called balanced case, when $e^+ \sim e^-$, and we can therefore represent the Fourier energy spectra in the form

$$e^\pm(k_\parallel, k_\perp) = f^\pm(k_\parallel/k_\perp) k_\perp^{-\alpha}, \quad (2)$$

where $f^\pm(x)$ do not vary significantly for $x < C$ and decline fast for $x > C$, with some constant C . Here k_\parallel is the wavevector in the direction of the uniform field \mathbf{B}_0 , and \mathbf{k}_\perp is the wavevector in the field-perpendicular direction. This form of the spectral functions is motivated by the fact that the energy redistribution occurring due to small-scale fluctuations (large k) is predominantly normal to the direction of the *local* guide field, which is the field produced by large-scale fluctuations. Therefore, compared to the direction of the *global* uniform field, the energy spectrum is smeared inside the small angle $\theta_0 \sim b_{rms}/B_0$, which implies a wedge-shaped energy-containing domain $k_\parallel < \theta_0 k_\perp$, or, the spectral function (2) with $C \sim \theta_0$, (e.g., Cho & Vishniac 2000; Maron & Goldreich 2001; Chen et al. 2011b). One can then write down a model equation for the spectral function (2), using certain closure assumptions. Differentiating $e^\pm(\mathbf{k})$ with respect to time, iterating equation (1) once, and splitting the forth-order correlation functions of \mathbf{z} 's into pair-wise correlations using Gaussian rule, we get¹

$$\partial_t e^\pm(k_\parallel, k_\perp) = \int M(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \Theta^\pm(k_{2\parallel}, k_{2\perp}) e^\mp(k_{2\parallel}, k_{2\perp}) [e^\pm(k_{1\parallel}, k_{1\perp}) - e^\pm(k_\parallel, k_\perp)] \times$$

¹Some extra assumptions are made in obtaining this equation, for instance it is assumed that the cross-correlation $\langle \mathbf{z}^+ \cdot \mathbf{z}^- \rangle$ is absent, see, e.g., Goldreich & Sridhar (1995). It should however be borne in mind that this equation is not rigorously derived from (1); it should be considered only as a model equation or as a plausible two-point closure.

$$\times \delta(k_{\parallel} - k_{1\parallel} - k_{2\parallel}) \delta(\mathbf{k}_{\perp} - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) d^3 k_1 d^3 k_2. \quad (3)$$

In this equation, the kernel has the form $M_{\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2} \propto (\mathbf{k}_{\perp} \times \mathbf{k}_{2\perp})^2 (\mathbf{k}_{\perp} \cdot \mathbf{k}_{1\perp})^2 / (k_{\perp}^2 k_{1\perp}^2 k_{2\perp}^2)$, and the Θ^{\pm} functions depend on the assumptions about the nonlinear interaction made in the model. In general we argue that these functions should concentrate in the region where the nonlinear interaction is essential, and inside this region they should scale as the inverse time of nonlinear interaction, that is, $\Theta^{\pm}(k_{\parallel}, k_{\perp}) \propto 1/\tau(k_{\perp})$. This can be summarized as follows, $\Theta^{\pm}(k_{\parallel}, k_{\perp}) = g^{\pm}(k_{\parallel}, k_{\perp}) k_{\perp}^{-\delta}$, where $g^{\pm}(k_{\parallel}, k_{\perp}) \approx \text{const}$ in the region of nonlinear interaction, and the nonlinear interaction time scales as $\tau(k_{\perp}) \propto k_{\perp}^{\delta}$.

To understand better our model (3), consider particular examples. In the case of weak turbulence, $g(k_{\parallel}, k_{\perp}) \approx \text{const}$ in a quite narrow region $k_{\parallel} V_A \leq 1/\tau(k_{\perp})$ compared with the k_{\parallel} -widths of the functions e^{\pm} , and it declines fast for $k_{\parallel} V_A \geq 1/\tau(k_{\perp})$. Therefore, $\Theta(k_{\parallel}, k_{\perp})$ is a broadened delta-function of k_{\parallel} , obeying $\int \Theta(k_{\parallel}, k_{\perp}) dk_{\parallel} = \text{const}$. One then recovers the theory by Galtier et al. (2000). In the case of strong turbulence, one expects the nonlinear interaction to be important in the same region where the energies e^{\pm} are concentrated, that is, $g^{\pm} \approx \text{const}$ in the region $k_{\parallel} \leq \theta_0 k_{\perp}$, it declines outside of this region, and $\tau \sim \lambda/z(\lambda)$. This way we recover the Goldreich & Sridhar (1995) theory. If in addition to the assumptions of the GS theory, one assumes that there is persistent dynamic angular alignment between magnetic and velocity fluctuations, which reduces the nonlinear interaction by $\theta_{\lambda} \sim \lambda^{1/4}$, one needs to multiply the kernel in (3) by $\theta_{\lambda}^2 \sim k_{\perp}^{-1/2}$, and assume that $\tau \sim \lambda/(v_{\lambda} \theta_{\lambda})$. One then recovers the theory by Boldyrev (2006). In view of this, we stress that model (3) provides a useful description of the spectral energies in MHD turbulence, however, it crucially depends on the scaling assumptions about the interaction time, incorporated in the model.²

The steady spectrum of turbulence can then be found by requiring that the collision integral in the rhs of (3) is zero. This leads to the spectrum of weak turbulence (Ng & Bhattacharjee 1996; Galtier et al. 2000): $E^{\pm}(k_{\parallel}, k_{\perp}) = e^{\pm}(k_{\parallel}, k_{\perp}) 2\pi k_{\perp} \propto f_w^{\pm}(k_{\parallel}) k_{\perp}^{-2}$, where $f_w^{\pm}(k_{\parallel})$ depend on the details of large-scale driving. The field-perpendicular spectrum of strong turbulence in Goldreich & Sridhar (1995) theory is then found as $E^{\pm}(k_{\perp}) = \int e^{\pm}(k_{\parallel}, k_{\perp}) 2\pi k_{\perp} dk_{\parallel} \propto k_{\perp}^{-5/3}$, while the spectrum in Boldyrev (2006) theory is $E^{\pm}(k_{\perp}) = \int e^{\pm}(k_{\parallel}, k_{\perp}) 2\pi k_{\perp} dk_{\parallel} \propto k_{\perp}^{-3/2}$. Numerical simulations do produce the spectrum k_{\perp}^{-2} for weak turbulence (Perez & Boldyrev 2008), and the spectrum $k_{\perp}^{-3/2}$ for strong turbulence with a strong guide field \mathbf{B}_0 , e.g., (Müller & Grappin 2005; Mason et al. 2008).

3. The spectrum of the residual energy

Recently, it has been realized that significant role in turbulence dynamics is played by the so-called residual energy, that is, the energy difference between magnetic and kinetic fluctuations, $E_r = E_v - E_b$, see (Boldyrev & Perez 2009; Wang et al. 2011; Boldyrev et al. 2011). Indeed, the complete description of the second-order statistics of two fluctuating fields, \mathbf{v} and \mathbf{b} requires *three* independent correlation functions. Two of them are provided by the autocorrelation functions of the Elsasser variables, that is, the

²The same statement is true for the so-called EDQNM closures often used to derive equations of type (3) for the spectra of strong turbulence. While providing physically reasonable models of turbulence, such equations are not derived from first principles and they crucially depend of model assumptions.

energy spectra (2). The third one, the cross-correlation function, is the residual energy $E_r = \int (\mathbf{z}^+ \cdot \mathbf{z}^-) d^3x = \int \text{Re}[e^r(\mathbf{k})] d^3k$, where, by definition, $e^r(\mathbf{k}) = \mathbf{z}^+(\mathbf{k}) \cdot \mathbf{z}^{-*}(\mathbf{k})$. It is easy to see that $e^r(\mathbf{k})$ is a complex function, while the residual energy spectrum is its real part.

The residual energy has been previously addressed in the literature (e.g., Pouquet et al. 1976; Grappin et al. 1983; Zank et al. 1996; Müller & Grappin 2005; Ng & Bhattacharjee 2007; Chen et al. 2011a), but it has been studied to a much lesser extent compared to the Elsasser energies, possibly because it is not a conserved quantity, it is not sign-definite, and it does not exhibit a cascade in a turbulent regime. As a result, it cannot be expressed through the conserved Elsasser energies (or, equivalently, through the total energy or cross-helicity), which are commonly used in theoretical models and measured in observations. In fact, in many studies of MHD turbulence the residual energy is explicitly or implicitly assumed to be zero, see e.g., (Galtier et al. 2000). In this section we propose a model for the residual energy, analogous to Eq. (3). We demonstrate that in contrast with the scaling of the Elsasser energies, the scaling of the residual energy is quite robust, that is, it depends to a much lesser extent on the arbitrary scaling assumptions incorporated in the model.

To obtain the equation for the residual energy, we first note that in the absence of the nonlinear interaction, the spectral residual energy $e^r(\mathbf{k}) = \mathbf{z}^+(\mathbf{k}) \cdot \mathbf{z}^{-*}(\mathbf{k})$ oscillates in time, since $\mathbf{z}^+(\mathbf{k}) \propto \exp(ik_{\parallel}v_A t)$ and $\mathbf{z}^-(\mathbf{k}) \propto \exp(-ik_{\parallel}v_A t)$. When the nonlinear interaction is present, the residual-energy evolution equation should contain the terms describing interaction of the residual energy with the Elsasser fields, $\sim e^r e^{\pm}$, and generation of the residual energy by the Elsasser fields, $\sim e^+ e^-$. It has been recently realized that the terms describing the generation of the residual energy by the Elsasser fields are essentially nonzero (Wang et al. 2011), meaning that residual energy is spontaneously generated by turbulent dynamics even if it is zero initially. We start our discussion of the residual energy with more detailed consideration of these terms.

It is crucial to note that the terms describing generation of the residual energy by the Elsasser fields should have the same *dimension* as the rhs of Eq. (3). Indeed, the residual energy has the same dimension as the Elsasser energies, and it is generated due to same nonlinear interactions. We however do not need the exact structure of this term, rather, we need to know its *scaling* with respect to the wavenumber. It turns out that this scaling is rather universal. Indeed, the term in the rhs of Eq. (3) describes constant flux J of the Elsasser energies $e^{\pm}(\mathbf{k}_{\perp})$ in the field-perpendicular direction, that is, it should scale as $\frac{1}{k_{\perp}} \frac{\partial}{\partial k_{\perp}} J \propto k_{\perp}^{-2}$, no matter what particular model of turbulence is assumed. The term describing generation of the residual energy should then have the same scaling, although its structure is different. We therefore model the residual-energy generating term as $\alpha(k_{\parallel}, k_{\perp})$ with the only requirement that it is concentrated in the region where the Elsasser energies are concentrated, and it obeys $\int \alpha(k_{\parallel}, k_{\perp}) dk_{\parallel} \propto k_{\perp}^{-2}$.

The term describing relaxation of the residual energy due to its interaction with the Elsasser energies can be generally modeled as $-\gamma(k_{\parallel}, k_{\perp})e^r(k_{\parallel}, k_{\perp})$, where the relaxation rate γ depends on the spectrum of the Elsasser fields, and it is concentrated in the region where the nonlinear interaction is present. We now collect all the three terms to formulate our model equation for the residual energy:

$$\partial_t e^r(k_{\parallel}, k_{\perp}) = 2ik_{\parallel}v_A e^r - \gamma(k_{\parallel}, k_{\perp})e^r + \alpha(k_{\parallel}, k_{\perp}). \quad (4)$$

4. Discussion

We now apply our formalism to the cases of weak and strong turbulence. For weak turbulence, the energy cascades predominantly in the field-perpendicular direction for each k_{\parallel} , so that the field-parallel structure of the spectrum does not change with k_{\perp} . Moreover, in this case the residual-energy generating term can be shown to be negative (Wang et al. 2011). We can therefore write (restoring the dimensional coefficients) that $\alpha(k_{\parallel}, k_{\perp}) = -\alpha_w(v_{rms}^4/v_A)k_{\perp}^{-2}$ for $k_{\parallel} < k_{0\parallel}$, where α_w is a dimensionless constant and $k_{0\parallel} \sim 1/L_{\parallel}$ is the field-parallel spectral width of the Elsasser fields. Weak turbulence theory also allows one to estimate the time of nonlinear interaction of the fields, which gives $\gamma = \beta(v_{rms}^2/v_A)k_{\perp}$ for $k_{\parallel} \approx 0$ (it will be clear momentarily why only the region $k_{\parallel} \approx 0$ is relevant here), and β is a dimensionless constant. Solving Eq. (4) for long times, we get:

$$e_v - e_b = Re[e^r(k_{\parallel}, k_{\perp})] = -\frac{\alpha_w \beta v_{rms}^2 \epsilon^4 k_{\perp}^{-1}}{\beta^2 k_{\perp}^2 \epsilon^4 + 4k_{\parallel}^2}, \quad (5)$$

where $\epsilon = v_{rms}/v_A \sim b_{rms}/B_0 \ll 1$. The residual energy is concentrated in a narrow region around $k_{\parallel} \leq \beta \epsilon^2 k_{\perp}/2$, in agreement with previous findings (Boldyrev & Perez 2009; Wang et al. 2011). The phase-volume compensated field-perpendicular spectrum of the residual energy then has the structure

$$E_r(k_{\parallel}, k_{\perp}) = Re[e^r(k_{\parallel}, k_{\perp})] 2\pi k_{\perp} = -f_w(k_{\parallel}/k_{\perp}) k_{\perp}^{-2}, \quad (6)$$

where $f_w(x) = v_{rms}^2 \alpha_w \beta \epsilon^4 / (\beta^2 \epsilon^4 + 4x^2)$, as follows from (5). We can also define the field-perpendicular spectrum

$$E_r(k_{\perp}) = \int E_r(k_{\parallel}, k_{\perp}) dk_{\parallel} = \alpha_w \pi v_{rms}^2 \epsilon^2 k_{\perp}^{-1} \sim -v_{rms}^2 \epsilon^2 k_{\perp}^{-1}. \quad (7)$$

For the case of strong turbulence, the spectra of $e^{\pm}(k_{\parallel}, k_{\perp})$ are concentrated in the region $k_{\parallel} \leq \theta_0 k_{\perp}$, and it is reasonable to assume that the function $\alpha(k_{\parallel}, k_{\perp})$ is concentrated in the same region. Restoring the dimensional parameters, one can therefore write $\alpha(k_{\parallel}, k_{\perp}) = -\alpha_s(v_{rms}^3/L_{\perp})(\theta_0 k_{\perp})^{-1} k_{\perp}^{-2}$ for a given k_{\parallel} inside the region $k_{\parallel} \leq \theta_0 k_{\perp}$, where α_s is a dimensionless parameter, and L_{\perp} is the integral field-perpendicular scale of the fluctuations. Note that the power of k_{\perp} is fixed by the requirement $\int \alpha(k_{\parallel}, k_{\perp}) dk_{\parallel} \propto k_{\perp}^{-2}$. One can also assume the power-law behavior for the relaxation rate, $\gamma(k_{\parallel}, k_{\perp}) = \gamma k_{\perp}^{\mu}$ within the same region, where γ is a dimensional parameter. It should be noted, however, that in contrast with the function $\alpha(k_{\parallel}, k_{\perp})$, whose scaling could be established on dimensional grounds, the scaling of the function $\gamma(k_{\parallel}, k_{\perp})$ cannot be easily derived. One can only argue that this relaxation rate should compete with the linear frequency only in the region where the Elsasser energies themselves are concentrated, that is, $\mu \leq 1$. It is interesting, however, that this bound is enough to establish the field-perpendicular spectrum of the residual energy. The solution of (4) takes the form

$$e_v - e_b = Re[e^r(k)] = \frac{\gamma k_{\perp}^{\mu}}{\gamma^2 k_{\perp}^{2\mu} + 4k_{\parallel}^2 v_A^2} \alpha(k_{\parallel}, k_{\perp}). \quad (8)$$

To find the field-perpendicular energy spectrum, one integrates this result over k_{\parallel} . One does not need however to integrate the function $\alpha(k_{\parallel}, k_{\perp})$, since the prefactor is a narrower function (a broadened δ -function, in fact). The integral over k_{\parallel} is then independent of γk^{μ} , and the result is:

$$E_r(k_{\perp}) = \int \text{Re}[e_r(k_{\parallel}, k_{\perp})] 2\pi k_{\perp} dk_{\parallel} \sim -v_{rms}^2 L_{\perp}^{-1} k_{\perp}^{-2}, \quad (9)$$

where we used $\theta_0 \sim v_{rms}/v_A$. This result is in agreement with numerical studies (e.g., Müller & Grappin 2005). It is also of interest to establish the value of μ . This can be inferred from numerical simulations if one evaluates $E_r(k_{\parallel} = 0, k_{\perp}) \propto k_{\perp}^{-2-\mu}$. Our simulations (that will be reported elsewhere) indicate that, quite interestingly, $\mu \approx 1$, which allows us to write the general expression for the residual energy in the form:

$$E_r(k_{\parallel}, k_{\perp}) = \text{Re}[e^r(k_{\parallel}, k_{\perp})] 2\pi k_{\perp} = -f_s(k_{\parallel}/k_{\perp}) k_{\perp}^{-3}, \quad (10)$$

where $f_s(x) \approx 1/\theta_0$ for $x < \theta_0$, and it declines for $x > \theta_0$. Relations (4), (6-7), and (9-10) are the main results of our work; they provide a model for residual energy observed in the solar wind and in numerical simulations.

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